

Asymptotics of Pattern Classes of Set Partition and Permutation d -tuple Avoidance

Benjamin Gunby

Harvard Department of Mathematics

June 2017

This research is joint work with Dömötör Pálvölgyi.

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Paper hopefully on arXiv soon!

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- We will call these sets *blocks* of the partition.
- Since the order of sets is irrelevant, we will order the B_i in increasing order of smallest element, and denote the set partition with slashes between the blocks. (e.g. $\{5, 2, 3\} \cup \{4, 6, 1\}$ is denoted 146/235).

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(Example: 146/235 contains the partition 12/34, as we can see by restricting 146/235 to the set $\{2, 3, 4, 6\}$.)

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- 3 There exists $d \in \mathbb{Z}^+$ and constants $c' > c > 0$ such that

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Question

Which pattern classes fall into which growth rates? That is, which d corresponds to a given \mathcal{P} ?

Definition

Given $n, d \in \mathbb{Z}^+$, and $\sigma_1, \dots, \sigma_d \in S_n$, we can construct a set partition of $[(d+1)n]$ as follows: there will be n blocks B_1, \dots, B_n , with $B_i = \{i, n + \sigma_1(i), 2n + \sigma_2(i), \dots, dn + \sigma_d(i)\}$. We call this set partition $[\sigma_1, \dots, \sigma_d]$.

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Example: $[132, 321] = 149/268/357$

Definition

Let π be a set partition. Then the permutability of π , denoted $pm(\pi)$ is the minimum positive integer d such that π is contained in a set partition of the form $[\sigma_1, \dots, \sigma_d]$, for some $m \in \mathbb{Z}^+$ and $\sigma_1, \dots, \sigma_d \in S_m$.

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Definition (Alternate)

The set partition π of $[n]$ has permutability d if and only if $[n]$ can be divided into $d + 1$ intervals such that each interval contains at most one element of each block of π , but it cannot be divided into d intervals in that way.

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Example: if $\mathcal{P} = Av(\pi)$, then $d = pm(\pi)$.

Theorem

$$c^n n^{(1-\frac{1}{d})n} < |\mathcal{P}_n| < c'^n n^{(1-\frac{1}{d})n},$$

where d is the smallest permutability not occurring in \mathcal{P} .

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This includes all set partitions of the form $[\sigma_1, \dots, \sigma_{d-1}]$, where $\sigma_i \in \mathcal{S}_{\frac{n}{d}}$, so at least

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$$\begin{aligned} & \left(\left(\frac{n}{d} \right)! \right)^{d-1} \\ & > \left(\frac{n}{ed} \right)^{\frac{d-1}{d}n}, \end{aligned}$$

proving the lower bound of the theorem.

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If \mathcal{P} is a pattern class not containing some π of permutability d , then $\mathcal{P} \subset Av(\pi) \subset Av([\sigma_1, \dots, \sigma_d])$ for some permutations $\sigma_1, \dots, \sigma_d$.

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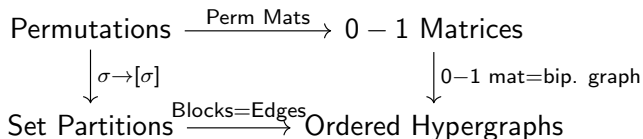
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Thus it suffices to show that $Av_n([\sigma_1, \dots, \sigma_d]) < c'^n n^{(1-\frac{1}{d})n}$ for some c' .

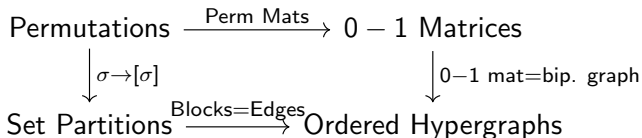
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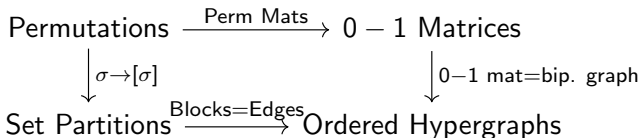


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Definition

A d -permutation hypergraph be the hypergraph corresponding to some set partition of the form $[\sigma_1, \dots, \sigma_{d-1}]$ (that is, the image of $[\sigma_1, \dots, \sigma_{d-1}]$ under the bottom map in the diagram above).

Hypergraph Avoidance

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For example, G contains the hypergraph H on $[4]$ with edges $\{1, 3\}$ and $\{2, 4\}$ if and only if there exist two *different* edges E_1 and E_2 of G and vertices $v_1, v'_1 \in E_1$, $v_2, v'_2 \in E_2$ with $v_1 < v_2 < v'_1 < v'_2$. (A single 4-vertex edge would not suffice.)

Main Lemma

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Lemma (G., Pálvölgyi)

Let H be a d -permutation hypergraph. Then there exists c such that any ordered hypergraph G on $[n]$ that avoids H has

$$\sum_{E \in E(G)} |E| < cn^{d-1}.$$

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- The case where G is also a d -permutation hypergraph is a theorem of Klazar-Marcus.

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In other words, a d -tuple T_1 contains another d -tuple T_2 if and only if each permutation in T_1 contains the corresponding permutation in T_2 at the same location.

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$\sigma_1 \leq \sigma_2$ in the Weak Bruhat Order.

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If $(\sigma'_1, \dots, \sigma'_d)$ avoids $(\sigma_1, \dots, \sigma_d)$, with $\sigma_1, \dots, \sigma_d \in S_n$, then $[\sigma'_1, \dots, \sigma'_d]$ is a set partition of $[(d+1)n]$ avoiding $[\sigma_1, \dots, \sigma_d]$.

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$$\begin{aligned} Av_n((\sigma_1, \dots, \sigma_d)) &\leq Av_{(d+1)n}([\sigma_1, \dots, \sigma_d]) \\ &< c'^n n^{n(1-\frac{1}{d})(d+1)} \\ &= c'^n n^{\left(\frac{d^2-1}{d}\right)n} \end{aligned}$$

for some c' .

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Put it all together:

Theorem (G., Pálvölgyi)

Let $\sigma_1, \dots, \sigma_d \in S_m$, $m > 1$. There exist constants $c' > c > 0$ such that

$$c^n n^{\binom{d^2-1}{d}} < Av_n((\sigma_1, \dots, \sigma_d)) < c'^n n^{\binom{d^2-1}{d}}$$

An Open Question

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For d -tuples, can obtain within an exponential factor of $n^{\alpha n}$, for $\alpha = d - \frac{1}{d_1} - \dots - \frac{1}{d_k}$, $d_i \in \mathbb{Z}^+$, $\sum d_i \leq d$. Can we obtain anything else?

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Can we compute the exponential factors in any of these cases?
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- For $Av((12, 12))$, we should have $c^n(n!)^{\frac{3}{2}}$.
- Exponential factor is not currently known!
- $\sqrt{\frac{\pi}{2}}$?

Thank You!