Open Problem: Can we count the equivalence classes under the $\{132, 231\}\{213, 312\}$ pattern-replacement equivalence?

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Pattern Replacement: A simple example

Pattern-replacement set: $\{abc, bca\}$

- $a < b < c$
- Letters in a permutation must be consecutive to form a pattern.

$$2\underbrace{134}_{abc} \;\text{———}\; 2\underbrace{341}_{bca}$$

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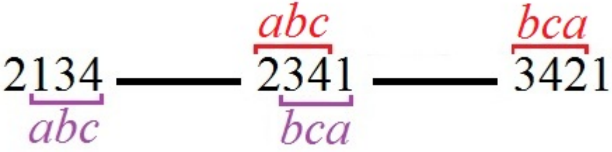
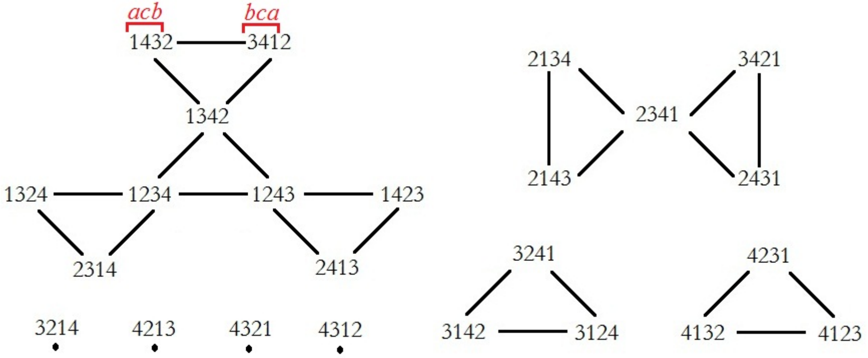


LOOKING AT A GRAPH

Pattern-replacement set: $\{abc, acb, bca\}$

Graph of all permutations of size 4 (S_4):



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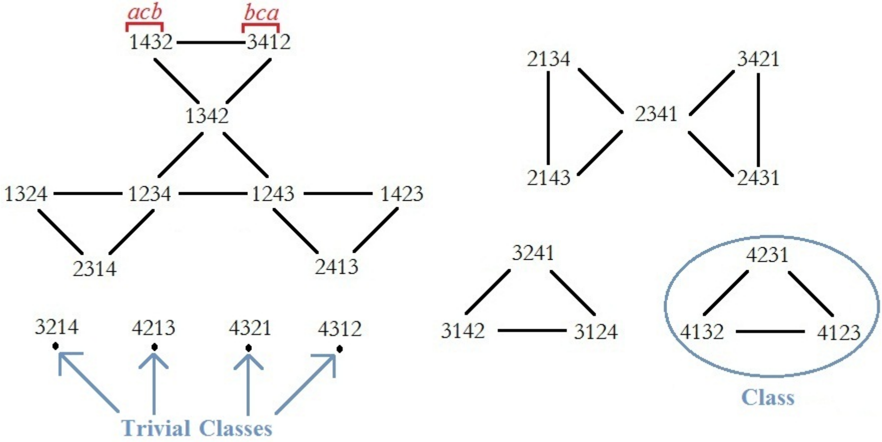

Two sets of two patterns are historically particularly interesting

- Donald Knuth: $\{bac, bca\}\{acb, cab\}$-equivalence, the Knuth relation
  - Number of classes in S_n: $f(n) = f(n-1) + (n-1) \cdot f(n-2)$
- Novelli and Schilling: $\{bac, acb\}\{bca, cab\}$-equivalence, the forgotten relation
  - Number of classes in S_n: $n^2 - 3n + 4$

D. Knuth, *Permutations, Matrices and Generalized Young tableaux*, Pacific J. Math. Volume 34, Number 3 (1970), 709-727.

J.-C. Novelli, A. Schilling, *The Forgotten Monoid*, RIMS Kokyuroku Bessatsu B8 (2008), 71-83.

Considering all pairs of sets of two patterns

Goal: For each choice of distinct $x, y, z, w \in S_3$, how many classes are there in S_n under $\{x, y\}\{z, w\}$-equivalence?

What's Known: All but one case solved in my paper *Counting Permutations Modulo Pattern-Replacement Equivalences for Three-Letter Patterns* (2014).

Open Question: How many equivalence classes are there in S_n under the $\{bac, cab\}\{acb, bca\}$-equivalence?

Computational Data:

n	3	4	5	6	7	8	9	10	11	12
# classes	4	10	26	76	234	782	2804	10972	47246	224648