Splittability and 1-Amalgamability of Permutation Classes

Michal Opler

In the study of permutation classes, a notable interest has recently been directed towards the operation of merging. We say that a permutation π is a *merge* of σ and τ if the elements of π can be colored red and blue so that the red elements form a copy of σ and the blue elements form a copy of τ.

A general problem that follows naturally is how to identify when a permutation class C has proper subclasses A and B, such that every element of C can be obtained as a merge of an element of A and an element of B. We say that such permutation class C is *splittable*. Jelínek and Valtr [1] mentioned the connection of splittability to more general structural properties of classes of relational structures studied in the area of Ramsey theory, most notably the notion of 1-amalgamability. We say that a permutation class C is *1-amalgamable* if given two permutations $\sigma, \tau \in C$, each with a marked element, we can find a permutation $\pi \in C$ containing both σ and τ such that the two marked elements coincide.

Not much is known about 1-amalgamability of permutation classes. Jelínek and Valtr [1, Lemma 1.5], using a more general result from Ramsey theory, showed that unsplittability implies 1-amalgamability, and they raised the question whether there is a permutation class that is both splittable and 1-amalgamable. In this talk, we answer this question by showing that the class $\mathrm{Av}(1423, 1342)$ has both properties.

Theorem 1. *The class $Av(1423, 1342)$ is both splittable and 1-amalgamable.*

For this task, we will introduce a slightly weaker property than being inflation-closed, that is closed under inflating just the elements that are left-to-right minima. We are motivated by the fact that $\mathrm{Av}(1423, 1342)$ can be obtained as a closure of a simple principal class under inflating left-to-right minima.

Proposition 2. *The class $Av(1423, 1342)$ is the closure of $Av(123)$ under inflating left-to-right minima.*

And finally we will show that certain properties of a permutation class C imply that its closure under inflating left-to-right minima is splittable and 1-amalgamable. These results may be of independent interest for the future study of splittability and 1-amalgamability.

This is joint work with Vít Jelínek.

References

[1] Vít Jelínek and Pavel Valtr. Splittings and Ramsey properties of permutation classes. *Adv. in Appl. Math.*, 63:41–67, 2015. ISSN 0196-8858.