PROLIFIC PERMUTATIONS AND PERMUTED PACKINGS

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The pattern poset $\mathcal{P}$ is a partial ordering of all permutations, by pattern containment. For example, $312 \prec 4132$ in $\mathcal{P}$. Certainly a permutation of n letters can cover at most n distinct permutations in $\mathcal{P}$, but certain degeneracies can reduce this number of covering relations. For example, because 413 and 412 are both 312-patterns in 4132, the permutation 4132 only covers three distinct elements in $\mathcal{P}$.

We say that permutation of n letters is *k-prolific* if each $(n-k)$-subset of the letters in its one-line notation forms a unique pattern. In other words, the permutation $w \in S_n$ is k-prolific if w has maximally many (that is, $\binom{n}{k}$) descendants k levels down in $\mathcal{P}$.

We present a complete characterization of k-prolific permutations for each k, proving that k-prolific permutations of m letters exist for every $m \geq k^2/2 + 2k + 1$, and that none exist of smaller size. In other words, the fewest letters needed to build a k-prolific permutation is $\lceil k^2/2 + 2k + 1 \rceil$, and this many letters will always suffice.

Key to these results is a natural bijection between k-prolific permutations and certain "permuted" packings of diamonds.

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