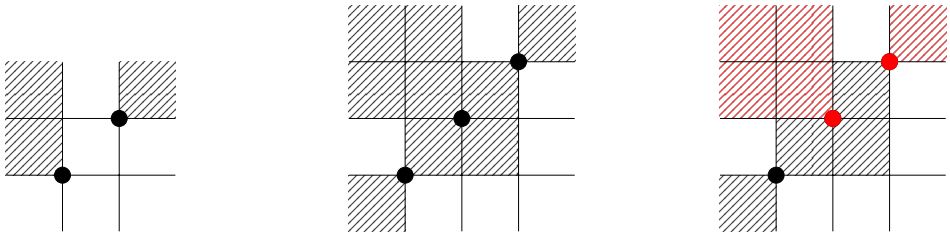
The Poset of Mesh Patterns

Jason P Smith (University of Strathclyde)
This talk is based on joint work with Henning Ulfarsson and Anders Claesson.

Mesh patterns are a generalisation of permutations and have been studied extensively in recent years. A natural definition of when one mesh patterns occurs in another mesh patterns was given in [TU17]. Therefore, we can also generalise the classical permutation poset to a poset of mesh patterns, where $(\sigma, S) \leq (\pi, P)$ if there is an occurrence of (σ, S) in (π, P).

To define when a mesh pattern occurs within another mesh pattern, first we need to recall when a mesh pattern (σ, S) occurs in a permutation π. Given any occurrence η of σ in π, in the classical permutation pattern sense, then each box (i, j) of (σ, S) corresponds to an area $R_\eta(i, j)$ in the plot of π. We say that η is an occurrence of the mesh pattern (σ, S) in the permutation π if there is no point in any of the areas $R_\eta(i, j)$ for any shaded box $(i, j) \in S$. We define an occurrence of a mesh pattern (σ, S) in another mesh pattern (π, P) as an occurrence η of (σ, S) in π where if (i, j) is shaded in (σ, S), then every box in $R_\eta(i, j)$ is shaded in (π, P).



(a) (b) (c)

Figure 1: Two mesh patterns (a) and (b), with (c) showing an occurrence of (a) in (b) in red.

The poset of mesh patterns has a very complex and interesting structure. We present some preliminary results, and open problems, on the poset. For example, the Möbius function is unbounded, but almost always zero, and the poset is not pure, that is, the maximal chains do not always have the same length. Moreover, the interval $[(\sigma, S), (\pi, P)]$ often has very different properties than the interval $[\sigma, \pi]$ in the classical permutation poset.

References

[TU17] Murray Tannock and Henning Ulfarsson. Equivalence classes of mesh patterns with a dominating pattern. *arXiv preprint arXiv:1704.07104*, 2017.