The Möbius function of increasing oscillations

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An interval $[\sigma, \pi]$ in a poset is the sub-poset defined as $\{\tau : \sigma \leq \tau \leq \pi\}$. The Möbius function $\mu[\sigma, \pi]$ is defined on an interval of a poset as follows: for $\sigma \not\leq \pi$, $\mu[\sigma, \pi] = 0$; for all λ, $\mu[\lambda, \lambda] = 1$; and for $\sigma < \pi$,

$$\mu[\sigma, \pi] = -\sum_{\sigma \leq \lambda < \pi} \mu[\sigma, \lambda].$$

The *increasing oscillating sequence* is the sequence $4, 1, 6, 3, 8, 5, \ldots, 2k+2, 2k-1, \ldots$. An *increasing oscillation* is a simple permutation contained in the increasing oscillating sequence.

Preliminary results

Extending results from Burstein, Jelínek, Jelínková and Steingrímsson [1], we show that if σ is sum indecomposable, then $\mu[\sigma, \pi]$ can be expressed as

$$\mu[\sigma, \pi] = -\sum_{\alpha \in \mathfrak{C}_{\sigma,\pi}} \mu[\sigma, \alpha] W(\sigma, \alpha, \pi).$$

where $\mathfrak{C}_{\sigma,\pi}$ is a subset of the sum indecomposable permutations found in π, and $W(\sigma, \alpha, \pi)$ is a $\{\pm 1, 0\}$-valued weighting function.

Increasing oscillations

If π is an increasing oscillation, then apart from the trivial cases, π is sum indecomposable.

We show that the calculation of the Möbius function where σ is an increasing oscillation can be performed without needing to evaluate any intermediate Möbius values $\mu[\sigma, \lambda]$, except those where $|\sigma| = |\lambda|$ or $|\sigma| = |\lambda| - 1$, where the values are $+1$ and -1 respectively.

We conjecture that if π is an increasing oscillation, then the absolute value of the principal Möbius function, $\mu[1, \pi]$, is asymptotically bounded by the interval $[0.6218n^2, n^2]$ if the length of π is even, and by the interval $[0.6218(n^2+n), n^2+n]$ if the length of π is odd.

*This talk is based on joint work with **Robert Brignall**.*

References

[1] A. Burstein, V. Jelínek, E. Jelínková, and E. Steingrímsson. The Möbius function of separable and decomposable permutations. *Journal of Combinatorial Theory, Series A*, 118(8):2346–2364, 2011.