SPLIT PATTERN AVOIDANCE (AFTER TIMOTHY ALLAND AND EDWARD RICHMOND)

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This talk is based on work of Timothy Alland and Edward Richmond (arXiv:1610.03535). Starting from a question in algebraic combinatorics inspired by a question in algebraic geometry, they define the notion of **split pattern avoidance** and **complete split pattern avoidance**. They then show the following:

Theorem 1. *A permutation w completely split avoids $3|12$ and $23|1$ if and only if w avoids 3412, 52341, and 635241.*

The main definitions are as follows:

Definition 2. A permutation $w \in S_n$ **split contains the split pattern** $u(1)\cdots u(j)|u(j+1)\cdots u(k)$ **with respect to position** r if and only if there exist $i_1 < \cdots < i_k$ such that w contains u at positions $i_1 < \cdots < i_k$ and $i_j \leq r < i_{j+1}$. We say w split avoids the split pattern with respect to r otherwise.

This means the pattern is divided into two halves, and the halves must embed into the permutation on opposite sides of position r. Clearly, w avoids u if and only if it split avoids $u(1)\cdots u(j)|u(j+1)\cdots u(k)$ with respect to all positions r.

Definition 3. A permutation $w \in S_n$ **completely split avoids** a set of split patterns $S = \{u_1(1)\cdots u_1(j_1)|u_1(j_1+1)\cdots u_1(k_1), \ldots, u_m(1)\cdots u_m(j_m)|u_m(j_m+1)\cdots u_m(k_m)\}$ if either $w = 1$, or if there exists a position r (with $1 \leq r \leq n-1$) such that w split avoids all the split patterns in S AND $w(1)\cdots w(r)$ and $w(r+1)\cdots w(n)$ both completely split avoid S.

We have the following lemma:

Lemma 4. A permutation w completely split avoids a set S if and only if w avoids every permutation $v \in S_m$ such that v contains one of the split patterns in S with respect to every position r (with $1 \leq r \leq m-1$).

A case-by-case argument then shows the following:

Proposition 5. *If v split contains either $3|12$ or $23|1$ with respect to every position r, then v contains 3412, 52341, or 635241.*

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